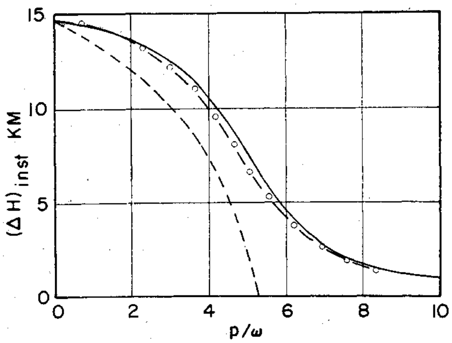

Fig. 1 Plot of stability boundaries [Eqs. (10) and (14)] for numerical example of Ref. 1, compared with exact calculation; --- criterion of Ref. 1, —— Eq. (14), O-O-O exact.

where $\omega^2 = -(\frac{1}{2})\rho SlV^2I^{-1}C_{M\alpha}$. This relation was then checked by the numerical integration of the exact equations for a particular missile with an initial angle of zero and a unit angular velocity. For these conditions

$$K_1(0) = K_2(0) \tag{12}$$

The maximum resultant angle of attack occurs when the modal amplitudes add;

$$|\zeta|_{\max} = K_1 + K_2 \tag{13}$$

where

$$K_j = K_j^{(0)} \exp\left\{\int_0^t \hat{\lambda}_j dt\right\}$$

Ohman determines the onset of instability for a given spin rate as the altitude above the initial point (ΔH) at which $|\zeta|_{\max}$ starts to grow and compares that with the altitude for which the larger $\hat{\lambda}_j$ changes sign. But from Eq. (13) we see that the modal amplitude for the smaller $\hat{\lambda}_j$ will continue to shrink after the other amplitude starts to grow and, therefore, $|\zeta|_{\max}$ will initially continue to shrink. Thus, the exact calculations should be compared with the altitude for minimum $|\zeta|_{\max}$. This can be obtained by differentiating Eq. (13);

$$K_1\hat{\lambda}_1 + K_2\hat{\lambda}_2 = 0 \tag{14}$$

Equation (14) is an implicit function of altitude and spin and can be easily solved for a given spin. In Fig. 1, the stability boundaries given by Eqs. (10) and (14) are plotted for the numerical example of Ref. 1 and compared with the exact calculation. The excellent agreement of the exact result with Eq. (14) shows the validity of Eqs. (4–5).

References

[1] Ohman, L. H., "A Spin-Yaw Instability Criterion for the Ascending Ballistic Missile," *Journal of Spacecraft and Rockets*, Vol. 4, No. 10, Oct. 1967, pp. 1374–1376.

[2] Murphy, C. H., "Free Flight Motion of Symmetric Missiles," BRL Rept. 1216, AD 645284, July 1963, Ballistic Research Labs.

[3] Murphy, C. H., "Angular Motion of a Re-Entering Symmetric Missile," *AIAA Journal*, Vol. 3, No. 7, July 1965, pp. 1275–1282.

Reply by Author to C. H. Murphy

L. H. Ohman*
National Research Council, Ottawa, Canada

In Ref. 1 Murphy considers the work of Ref. 2 by the present author in order to clear a misconception occurring in the latter work. This is gratefully acknowledged. Furthermore, Murphy derives the stability criterion under more rigorous assumptions than was done in Ref. 2. Ohman uses a "mode separation method" and considers the stability of the presumed most unstable mode separately and arrives at an explicit form of the stability criterion, whereas Murphy considers the stability of the coupled system and arrives at an implicit form of the stability criterion. It is obvious then that the stability prediction á la Murphy will agree better with numerical results than those á la Ohman. However, there is always a merit in having expressions in explicit form and this may justify some sacrifices in accuracy.

Murphy is right that the sign of the Magnus moment determines which mode will have less damping. However, employing the mode separation method for stability analysis, one finds that in either case it is the less damped mode that first becomes unstable. It follows then that the sign of the Magnus moment is irrelevant with regard to the stability of the system, at least for similar initial conditions in both modes and the criterion of Ref. 2 is thus still valid. This has been verified by solving the full equations for the case $p = 37$ rad/sec with the sign of $C_{m_{p\beta}}$ reversed. The point of instability was not affected by the sign of $C_{m_{p\beta}}$, although there was a slight difference in the actual motion; for $t = 10$ sec the value of $\alpha/\alpha(0)$ was 0.02 higher for $+C_{m_{p\beta}}$ than for $-C_{m_{p\beta}}$.

Finally, the author wants to make it clear that the work in Ref. 2 is related to sounding rockets for which in general $I_x/I \ll 1$ and the roll rate p is low. For spinning mortar shells, e.g., the assumptions regarding I_x/I and p no longer hold and the conclusions regarding the Magnus moment of Ref. 2 are invalid.

References

[1] Murphy, C. H., "Stability Criterion for Ascending Ballistic Missiles," *Journal of Spacecrafts and Rockets*, Vol. 5, No. 4, April 1968, pp. 495–496.

[2] Ohman, L. H., "A Spin-Yaw Instability Criterion for the Ascending Ballistic Missile," *Journal of Spacecraft Rockets*, Vol. 4, No. 10, Oct. 1967, pp. 1374–1376.

Received February 1, 1968.

* Associate Research Officer, High Speed Aerodynamics National Aeronautical Establishment.